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Slender-ship shallow-water flow past a slowly varying bottom

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SUMMARY

The unsteady subcritical potential flow of a slender ship moving past a slowly varying bottom in shallow water is analyzed using the methods of matched asymptotic expansions and multiple scales. The hydrodynamic pressure field on the ship is obtained to second order in the slenderness parameter.

1. Introduction

Following the work of Tuck [1], a number of papers have appeared in the literature which analyze the potential-flow hydrodynamics of a slender ship moving in shallow water using the method of matched asymptotic expansions. Among those that consider water of constant depth are Tuck [2], Newman [3], Tuck [4], Taylor and Tuck [5] and Lea and Feldman [6]. Plotkin [7, 8] studied the steady flow past an anchored ship in water of variable depth and then, in [9, 10], analyzed the unsteady subcritical flow due to a slender ship moving past a wavy wall. In [9], the wall wavelength was the same order as the ship length, and in [10], the wall wavelength was smaller than the ship length but larger than the transverse dimensions of the ship. In the latter problem, the method of multiple scales was introduced to handle the dual characteristic lengths in the streamwise direction. In both of the wavy-wall cases, the bottom location varied only slightly from its mean position.

In this paper, the unsteady subcritical potential flow due to a slender ship translating in shallow water with arbitrary slowly varying depth variation in the streamwise direction is analyzed. The analysis follows closely that of Tuck [1] for the solution of the constant-depth problem.

2. Problem formulation

Consider a slender ship of length $2l$ translating with constant speed U in shallow water. A Cartesian coordinate system is introduced with its origin fixed to the ship at midship. z is measured upward from the undisturbed free surface and x is opposite to the direction of motion. The system is shown in Figure 1. The beam and draft are small, of $O(\varepsilon)$, with respect to the length, and the hull surface is given by $y = \varepsilon f(x, z)$ in the moving system. For the water to be considered shallow, the water depth must also be of $O(\varepsilon)$ and the bottom is given by $z = -h(x - Ut, y)$. The depth Froude number, $F = U/(gh)^{\frac{1}{2}}$, is taken to be of $O(1)$ and is less than one.

For incompressible irrotational flow, the velocity is represented as the positive gradient of a

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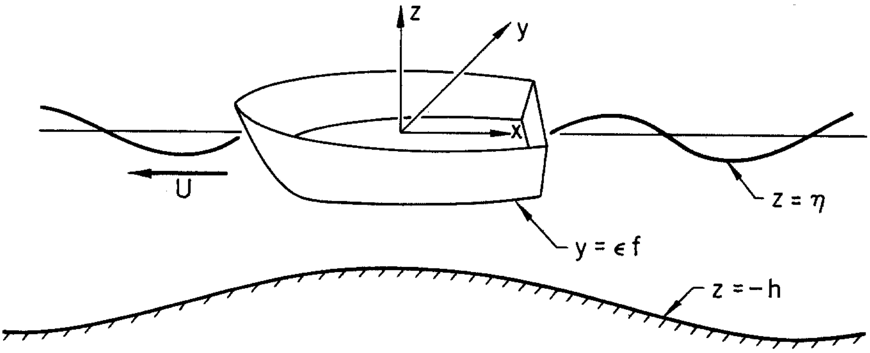

Figure 1. Coordinate system.

velocity potential $\varphi(x, y, z, t)$ which satisfies Laplace's equation. The complete set of equations is given in Plotkin [9] as

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0, \text{ in fluid domain,} \tag{2.1}$$

$$\varepsilon U f_x + \varepsilon \phi_x f_x + \varepsilon \phi_z f_z - \phi_y = 0, \text{ on } y = \varepsilon f, \tag{2.2}$$

$$\phi_z + \phi_x h_x + \phi_y h_y = 0, \text{ on } z = -h, \tag{2.3}$$

$$\eta_t + U\eta_x + \phi_x \eta_x + \phi_y \eta_y - \phi_z = 0, \text{ on } z = \eta, \tag{2.4}$$

$$\phi_t + U\phi_x + (\phi_x^2 + \phi_y^2 + \phi_z^2)/2 + gz = 0, \text{ on } z = \eta, \tag{2.5}$$

where $z = \eta(x, y, t)$ describes the free surface and g is the gravitational acceleration.

The bottom is taken to vary slowly in the streamwise direction with a length scale of $O(\varepsilon^{-1})$. Its description is therefore

$$z = -h[\varepsilon(x - Ut)]. \tag{2.6}$$

It is noted that now there are two characteristic length scales in the streamwise direction – the ship length of $O(1)$ and the scale of the bottom variation of $O(\varepsilon^{-1})$. Also, a slow time scale of $O(\varepsilon^{-1})$ is introduced via equation (2.6). The method of multiple scales (Nayfeh [11]) is used. Let

$$X_1 = x, \ X_2 = \varepsilon x, \quad T_1 = t, \ T_2 = \varepsilon t, \tag{2.7}$$

and

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial X_1} + \varepsilon \frac{\partial}{\partial X_2}, \quad \frac{\partial}{\partial t} = \frac{\partial}{\partial T_1} + \varepsilon \frac{\partial}{\partial T_2}. \tag{2.8}$$

The velocity potential depends independently on the variables in equation (2.7).

Since ε is an appropriate scale in the vertical direction, the coordinate $Z = z/\varepsilon$ is used. The solution is then of the form

$$\phi = \phi(X_1, X_2, y, Z, T_1, T_2). \tag{2.9}$$

The method of matched asymptotic expansions is used to define the mathematical problems in the inner and outer regions following Tuck [1].

3. The inner expansion

The inner region, in the neighborhood of the ship, is defined by the following order of magnitude of the coordinates with respect to the ship length

$$x = O(1),\ y,\ z = O(\varepsilon). \tag{3.1}$$

Introduce the inner variable $Y = y/\varepsilon$. The velocity potential is expanded in an asymptotic series in ε of the form

$$\phi = \varepsilon\Phi^{(1)} + \varepsilon^2\Phi^{(2)} + \varepsilon^3\Phi^{(3)} + \dots. \tag{3.2}$$

Substitution into Laplace's equation yields

$$\Phi^{(1)}_{YY} + \Phi^{(1)}_{ZZ} = 0,\quad \Phi^{(2)}_{YY} + \Phi^{(2)}_{ZZ} = 0,\quad \Phi^{(3)}_{YY} + \Phi^{(3)}_{ZZ} = -\Phi^{(1)}_{X_1X_1}. \tag{3.3}$$

The boundary conditions on the hull and bottom, equations (2.2, 2.3), become

$$\Phi^{(1)}_N = 0,\quad \Phi^{(2)}_N = Uf_{X_1}/(1+f_Z^2)^{\frac{1}{2}},\quad \Phi^{(3)}_N = \Phi^{(1)}_{X_1}f_{X_1}/(1+f_Z^2)^{\frac{1}{2}},\qquad \text{on } Y=f, \tag{3.4}$$

and

$$\Phi^{(1)}_Z = \Phi^{(2)}_Z = \Phi^{(3)}_Z = 0,\ \text{on } Z = -h/\varepsilon, \tag{3.5}$$

where N is the normal in the cross-flow plane expressed in inner variables. The free-surface conditions, equations (2.4, 2.5), are combined and transfered to $Z = 0$ to become

$$\Phi^{(1)}_Z = 0 = \Phi^{(2)}_Z,\quad \Phi^{(3)}_Z = -(\Phi^{(1)}_{T_1T_1} + 2U\Phi^{(1)}_{X_1T_1} + U^2\Phi^{(1)}_{X_1X_1})/g\varepsilon. \tag{3.6}$$

Note that $U^2/g = O(\varepsilon)$. The time dependence in the solution is being driven by the bottom with T_2 as the appropriate time scale. In fact, to $O(\varepsilon^2)$ in the inner problem, the only explicit appearance of time in the governing equations is through the depth $h(X_2, T_2)$. It will therefore be assumed that $\Phi^{(1)}$ and $\Phi^{(2)}$ are independent of T_1.

$\Phi^{(1)}$ satisfies Laplace's equation in the cross-flow plane with zero normal derivative on all boundaries in this plane. Therefore

$$\Phi^{(1)} = \Phi^{(1)}(X_1, X_2, T_2), \tag{3.7}$$

and is arbitrary. The solution for $\Phi^{(2)}$ can be written as

$$\Phi^{(2)} = \Phi^{(21)}(X_1, X_2, T_2) + \Phi^{(22)}(Y, Z; X_1, X_2, T_2), \tag{3.8}$$

where $\Phi^{(21)}$ is arbitrary. A suitable boundary condition for $\Phi^{(22)}$ as $|Y| \to \infty$ is

$$\Phi^{(22)} \to u(X_1, X_2, T_2)\,|Y| + o(1). \tag{3.9}$$

u is determinable from conservation of mass as

$$u = U\varepsilon S_{X_1}(X_1)/2h(X_2, T_2), \tag{3.10}$$

where $\varepsilon^2 S$ is the hull cross-section area below the plane $Z = 0$.

The free-surface condition for $\Phi^{(3)}$ becomes

$$\Phi_Z^{(3)} = U^2 \Phi_{X_1X_1}^{(1)}/g\varepsilon, \tag{3.11}$$

and $\Phi^{(3)}$ is also taken to be independent of T_1.

Following Tuck [1], the solution for $\Phi^{(3)}$ can be written as

$$\Phi^{(3)} = \Phi_{X_1}^{(1)}\Phi^{(22)} + \Phi^{(31)}(X_1, X_2, T_2) + \Phi^{(32)}(Y, Z; X_1, X_2, T_2)$$
$$- \tfrac{1}{2}\Phi_{X_1X_1}^{(1)}[(1 - F^2)Y^2 + (Z + h/\varepsilon)^2 F^2]. \tag{3.12}$$

The first three terms individually satisfy Laplace's equation. The first term satisfies the hull boundary condition. $\Phi^{(31)}$ is arbitrary. The last term satisfies the Poisson equation, (3.3), and the free-surface condition, (3.11). (It is noted that the solution in Tuck [1] has an error in its equation (4.14). Fortunately, no conclusions are drawn from it by Tuck. The last term in Eq. (3.12) introduces a non-zero normal component of velocity at the hull which is not taken into account). $\Phi^{(32)}$ is introduced for this purpose and satisfies a homogeneous problem except for the hull boundary condition

$$\Phi_N^{(32)} = \Phi_{X_1X_1}^{(1)}[Y(1 - F^2) - f_Z F^2(Z + h/\varepsilon)]/(1 + f_Z^2)^{\frac{1}{2}}. \tag{3.13}$$

The behavior of $\Phi^{(32)}$ as $|Y| \to \infty$ can be determined from conservation of mass. The volume flux leaving the hull is

$$\int \Phi_N^{(32)} ds = \Phi_{X_1X_1}^{(1)}[S - U^2 B/g\varepsilon^2], \tag{3.14}$$

where the integral is taken around the wetted hull cross-section and $B(X_1)$ is the width of the cross-section at the waterline and is $O(\varepsilon)$. Therefore, as $|Y| \to \infty$

$$\Phi^{(32)} \to v(X_1, X_2, T_2)|Y| + o(1), \tag{3.15}$$

where v is the flux in equation (3.14) divided by $2h$.

4. The outer expansion

The outer region, far from the ship, is defined by the following order of magnitude of the coordinates

$$x, y = O(1), \quad z = O(\varepsilon). \tag{4.1}$$

The velocity potential is expanded in an asymptotic series in ε of the form

$$\phi = \varepsilon\phi^{(1)} + \varepsilon^2\phi \quad + \varepsilon^3\phi^{(3)} + \varepsilon^4\phi^{(4)} + \dots \tag{4.2}$$

Substitution into Laplace's equation yields

$$\phi^{(1)}_{ZZ} = \phi^{(2)}_{ZZ} = 0,\ \phi^{(3)}_{ZZ} = -\nabla^2\phi^{(1)},\ \phi^{(4)}_{ZZ} = -\nabla^2\phi^{(2)} - 2\phi^{(1)}_{X_1X_2}, \tag{4.3}$$

where $\nabla^2 = \partial^2/\partial X_1^2 + \partial^2/\partial y^2$.

The bottom boundary condition (2.3) becomes

$$\phi^{(1)}_Z = \phi^{(2)}_Z = \phi^{(3)}_Z = 0, \quad \phi^{(4)}_Z = -\phi^{(1)}_{X_1}h_{X_2}/\varepsilon. \tag{4.4}$$

Equations (4.3) are integrated once with respect to Z and using equations (4.4) and the information that matching with the inner expansion shows $\phi^{(1)}$ and $\phi^{(2)}$ to be independent of T_1, we get

$$\begin{aligned}
&\phi^{(1)} = \phi^{(1)}(X_1, y; X_2, T_2), \quad \phi^{(2)} = \phi^{(2)}(X_1, y; X_2, T_2),\\
&\phi^{(3)}_Z = -(Z + h/\varepsilon)\nabla^2\phi^{(1)},\\
&\phi^{(4)}_Z = -(Z + h/\varepsilon)(\nabla^2\phi^{(2)} + 2\phi^{(1)}_{X_1X_2}) - \phi^{(1)}_{X_1}h_{X_2}/\varepsilon.
\end{aligned} \tag{4.5}$$

The dynamic free-surface condition (2.5) suggests an expansion for η of the form

$$\eta = \varepsilon^2\eta^{(2)} + \varepsilon^3\eta^{(3)} + \ldots \tag{4.6}$$

where

$$\eta^{(2)} = -U\phi^{(1)}_{X_1}/g\varepsilon, \quad \eta^{(3)} = -[U\phi^{(2)}_{X_1} + \phi^{(1)}_{T_2} + U\phi^{(1)}_{X_2} + (\phi^{(1)2}_{X_1} + \phi^{(1)2}_y)/2]/g\varepsilon. \tag{4.7}$$

Substitution of equations (4.5–4.7) into the kinematic free-surface condition (2.4) yields

$$(1 - F^2)\phi^{(1)}_{X_1X_1} + \phi^{(1)}_{yy} = 0, \tag{4.3}$$

and

$$\begin{aligned}
(1 - F^2)\phi^{(2)}_{X_1X_1} + \phi^{(2)}_{yy} = {} & -2\phi^{(1)}_{X_1X_2} - h_{X_2}\phi^{(1)}_{X_1}/h + 2U[\phi^{(1)}_{X_1T_2} + U\phi^{(1)}_{X_1X_2}]/c^2\\
& + 2U[\phi^{(1)}_{X_1}\phi^{(1)}_{X_1X_1} + \phi^{(1)}_y\phi^{(1)}_{X_1y} + \phi^{(1)}_{X_1}\nabla^2\phi^{(1)}/2]/c^2,
\end{aligned} \tag{4.9}$$

where $c^2 = gh$.

Equations (4.8–4.9) will be solved formally using Green's function (source) distributions. The appropriate Green's function is

$$G(X_1, y) = (2\pi\beta)^{-1}\log(X_1^2 + \beta^2 y^2)^{\frac{1}{2}}, \tag{4.10}$$

where $\beta^2 = 1 - F^2$. Note that X_2 and T_2, the slow scales, are parameters in the differential equations entering through the Froude number F and depth h.

The solution for $\phi^{(1)}$ is

$$\phi^{(1)} = \int_{-\infty}^{\infty} \mu^{(1)}(\xi, X_2, T_2)G(X_1 - \xi, y)\,d\xi, \tag{4.11}$$

where the unknown source strength $\mu^{(1)}$ is allowed to be a function of X_2 and T_2. Since $\phi^{(1)}$ is a function of X_2 and T_2 only through the combination $X_2 - UT_2$, it is seen that the term proportional to $\phi^{(1)}_{X_1T_2} + U\phi^{(1)}_{X_1X_2}$ on the right-hand side of equation (4.9) is zero.

Let us now rewrite (4.9) as

$$(1-F^2)\phi^{(2)}_{X_1X_1} + \phi^{(2)}_{yy} = -2\phi^{(1)}_{X_1X_2} - h_{X_2}\phi^{(1)}_{X_1}/h + R(X_1, y; X_2, T_2), \tag{4.12}$$

where this equation defines R and R is seen to be the right-hand side for the constant-depth problem of Tuck [1]. The solution for $\phi^{(2)}$ is

$$\begin{aligned}\phi^{(2)} = {} & \int_{-\infty}^{\infty} \mu^{(2)}(\xi, X_2, T_2)G(X_1-\xi, y)d\xi \\ & - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [2\phi^{(1)}_{X_1X_2} + h_{X_2}\phi^{(1)}_{X_1}/h]G(X_1-\xi, y-\alpha)d\xi d\alpha \\ & + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} RG(X_1-\xi, y-\alpha)d\xi d\alpha.\end{aligned} \tag{4.13}$$

5. Matching

To determine the unknown functions $\mu^{(1)}$, $\mu^{(2)}$, $\Phi^{(1)}$ and $\Phi^{(21)}$, the inner and outer expansions must be matched. The following matching principle from Van Dyke [12] is used:

"The m-term inner expansion of the (n-term outer expansion) = the n-term outer expansion of the (m-term inner expansion)" (5.1)

Take $m = 3$ and $n = 2$. The two-term outer expansion is

$$\varepsilon\phi^{(1)} + \varepsilon^2\phi^{(2)}.$$

It has a three-term inner expansion of

$$\begin{aligned}&\varepsilon\phi^{(1)}(X_1, 0; X_2, T_2) + \varepsilon\mu^{(1)}|y|/2 - \varepsilon y^2(1-F^2)\phi^{(1)}_{X_1X_1}(X_1, 0; X_2, T_2)/2 \\ &\quad + \varepsilon^2\phi^{(2)}(X_1, 0; X_2, T_2) + \varepsilon^2\mu^{(2)}|y|/2.\end{aligned} \tag{5.2}$$

The three-term inner expansion is

$$\begin{aligned}&\varepsilon\Phi^{(1)} + \varepsilon^2[\Phi^{(21)} + \Phi^{(22)}] + \varepsilon^3\{\Phi^{(1)}_{X_1}\Phi^{(22)} + \Phi^{(31)} + \Phi^{(32)} - \Phi^{(1)}_{X_1X_1} \\ &\quad \times [(1-F^2)Y^2 + (Z+h/\varepsilon)^2F^2]\}.\end{aligned}$$

It has a two-term outer expansion of

$$\begin{aligned}&\varepsilon\Phi^{(1)} + \varepsilon u|y| - \varepsilon y^2(1-F^2)\phi^{(1)}_{X_1X_1}(X_1, 0; X_2, T_2)/2 \\ &\quad + \varepsilon^2\Phi^{(21)} + \varepsilon^2[u\Phi^{(1)}_{X_1} + v]|y|.\end{aligned} \tag{5.3}$$

Equating (5.2) and (5.3) we get

$$\begin{aligned}
&\Phi^{(1)} = \phi^{(1)}(X_1, 0; X_2, T_2), \quad \Phi^{(21)} = \phi^{(2)}(X_1, 0; X_2, T_2),\\
&\mu^{(1)} = 2u = U\varepsilon S_{X_1}(X_1)/h(X_2, T_2),\\
&\mu^{(2)} = 2v + 2u\Phi^{(1)}_{X_1}\\
&\quad = 2u\Phi^{(1)}_{X_1} + \Phi^{(1)}_{X_1X_1}[S(X_1) - U^2B(X_1)/g\varepsilon^2]/h(X_2, T_2).
\end{aligned} \tag{5.4}$$

To $O(\varepsilon^2)$, the velocity potential in the inner region is

$$\begin{aligned}
\phi = &\Bigg\{U\varepsilon^2(2\pi\beta h)^{-1}\int_{-l}^{l} S_{X_1}(\xi)\log|X_1 - \xi|\,d\xi\\
&+ \varepsilon^2\Bigg[\int_{-\infty}^{\infty}\mu^{(2)}(\xi, X_2, T_2)G(X_1 - \xi, 0)\,d\xi\\
&+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} RG(X_1 - \xi, -\alpha)\,d\xi\,d\alpha + \Phi^{(22)}\Bigg]\Bigg\}\\
&- \varepsilon^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[2\phi^{(1)}_{X_1X_2} + h_{X_2}\phi^{(1)}_{X_1}/h]G(X_1 - \xi, -\alpha)\,d\xi\,d\alpha.
\end{aligned} \tag{5.5}$$

The solution is seen to consist of two parts. The term in curly brackets is the constant-depth solution with the actual slowly varying values of depth h and Froude number F appearing. The second term is an $O(\varepsilon^2)$ term due to the depth variation and proportional to its streamwise slope h_{X_2}.

6. Pressure field in inner region

The hydrodynamic pressure is obtained from the Bernoulli equation

$$p = -\rho[\phi_t + U\phi_x + (\phi_x^2 + \phi_y^2 + \phi_z^2)/2], \tag{6.1}$$

where ρ is the fluid density. To $O(\varepsilon^2)$, the velocity potential in the inner region is

$$\phi = \varepsilon\Phi^{(1)} + \varepsilon^2[\Phi^{(21)} + \Phi^{(22)}]. \tag{6.2}$$

Substitution into (6.1) and use of inner variables and equation (2.8) leads to

$$p = -\rho U\varepsilon\Phi^{(1)}_{X_1} - \rho\varepsilon^2[\Phi^{(1)2}_{X_1}/2 + U\Phi^{(21)}_{X_1}] - \rho\varepsilon^2[U\Phi^{(22)}_{X_1} + (\Phi^{(22)2}_{Y} + \Phi^{(22)2}_{Z})/2]. \tag{6.3}$$

The first two terms are what Tuck [1] calls the "interaction" pressure since the cross-sectional area appears only inside of the Green's function integrals. The last term depends on the local behavior at each streamwise station and is cross-section dependent. If equation (5.5) is used, the pressure can be considered to consist, to $O(\varepsilon^2)$, of the constant-depth solution with the actual values of h and F and an additional term, proportional to h_{X_2}, *given by*

$$\rho U\varepsilon^2(2\pi\beta)^{-1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[2\phi^{(1)}_{X_1X_2} + h_{X_2}\phi^{(1)}_{X_1}/h]\frac{X_1 - \xi}{(X_1 - \xi)^2 + \alpha^2}\,d\xi\,d\alpha. \tag{6.4}$$

7. Discussion

The unsteady subcritical potential flow of a slender ship moving past a slowly varying bottom in shallow water is obtained to $O(\varepsilon^2)$ where ε is the slenderness parameter. The scale of the streamwise depth variation is $O(\varepsilon^{-1})$ and otherwise the depth variation is arbitrary.

It is seen that, to $O(\varepsilon)$, it is correct to represent the solution as the constant-depth solution if the actual values of depth and Froude number are used. This representation is incorrect to $O(\varepsilon^2)$ since a term proportional to the streamwise slope of the water depth appears in the solution. This term can be seen in the inner region velocity potential in equation (5.5) and pressure distribution in (6.4).

The solution for the slowly varying bottom differs significantly from the solution of Plotkin [9, 10] for the cases with bottom scale of $O(1)$ and $O(\varepsilon^{\frac{1}{2}})$, respectively. The time dependence here does not enter through the differential equation but enters implicitly through the depth dependence on the slow scale T_2. Time is, therefore, essentially a parameter of the solution.

Consider the first-order velocity potential in equation (5.5). It is the product of a term which varies with the fast scale, the ship length, and is dependent on the ship geometry, and a term, $(\beta h)^{-1}$, which varies with the slow scales, is dependent on the bottom geometry and essentially modulates the integral term.

8. Sample problem

Consider a hull of revolution with a parabolic waterline. The width and cross-sectional area are given by

$$B(x) = 2\varepsilon B_0(1 - x^2/l^2) \text{ and } S(x) = 0.5\pi B_0^2(1 - x^2/l^2)^2.$$

From equations (5.5 and 6.3), the pressure to $O(\varepsilon)$ in the inner region is (see [10])

$$p = \rho \frac{B_0^2 g\varepsilon^2}{2l} \frac{F^2}{(1-F^2)^{\frac{1}{2}}} \left[4\frac{x^2}{l^2} - \frac{8}{3} + \frac{x}{l}(1 - x^2/l^2)\cdot\log\left(\frac{x/l+1}{x/l-1}\right)^2\right] \tag{8.1}$$

To model a ship moving into water of decreasing depth, consider the linear depth distribution

$$h = h_0[1 + \varepsilon(x - Ut)/l].$$

Let F_0, the Froude number at $x = t = 0$, be 0.5 and let $\varepsilon = 0.2$. In Figure 2 the pressure along the ship (from equation (8.1)) is plotted for values of $Ut/l = 0$, 1.25 and 2.5 which correspond to depths at $x = 0$ of h_0, $3h_0/4$ and $h_0/2$. For comparison, the pressure corresponding to the constant depth h_0 is also plotted. For constant depth, the pressure is symmetric about $x = 0$.

For the unsteady case, the pressure at any streamwise station x varies with time through the function $F^2(1 - F^2)^{-\frac{1}{2}}$. This function increases monotonically with increasing F or decreasing h for $F < 1$. Therefore, for a given position along the ship the magnitude of the pressure increases monotonically with time for the linear depth variation. Also, since at any given time the depth increases linearly from the bow ($x/l = -1.0$) to the stern ($x/l = 1.0$), the pressure distribution will be asymmetric with the larger magnitudes on the forward half of the ship.

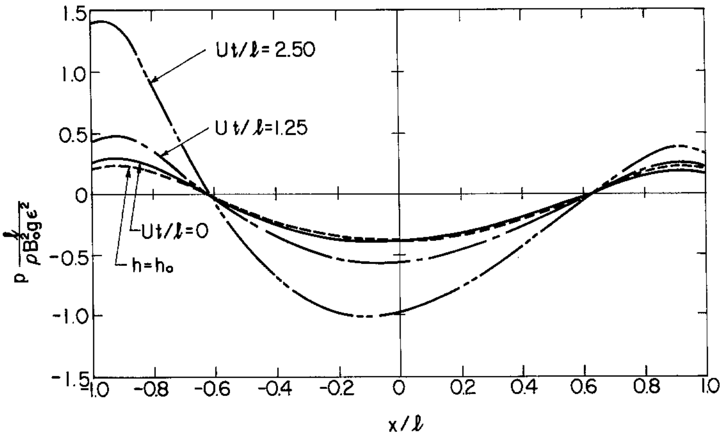

Figure 2. First-order pressure distribution for ship with parabolic waterline of beam $2\varepsilon B_0$, half-length l, with depth $h = h_0[1 + .2(x - Ut)/l]$ and Froude number based on h_0 of 0.5.

Acknowledgments

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